A Right Angle Theorem and the Equidistance Theorems

Advanced Geometry 4.3 and 4.4

Warm-Up (with a Partner)

Prove that if two angles are both supplementary and congruent, then they are right angles. (Write a paragraph proof).

Given: Diagram:

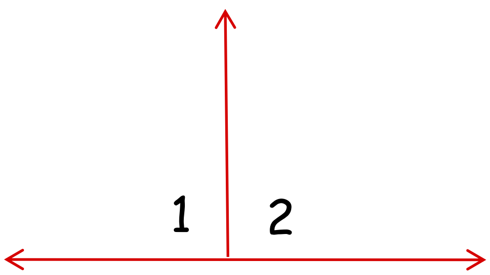
Prove:

Warm-Up (with a Partner)

Prove that if two angles are both supplementary and congruent, then they are right angles.

Given: $\angle 1 \cong \angle 2$

Diagram:



Prove: $\angle 1$ *and* $\angle 2$ are right angles

Since you can assume straight angles, angle 1 and angle 2 are supplementary. We are given that angle 1 and angle 2 are congruent, so if their sum is 180° and they have the same measure, they must be 90° or right angles.

You did it! Add to your Index Cards!

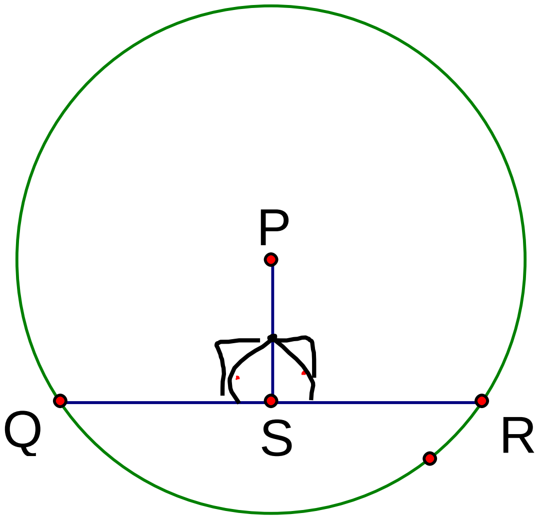
Right Angle Theorem:

Assumed

If two angles are both supplementary and congruent, then they are right angles.

CPCTC

Example



- Given: Circle P

  S is the midpoint of $\overline{QR}$

- Prove: $\overline{PS} \perp \overline{QR}$

Example

- Given: Circle P

  S is the midpoint of $\overline{QR}$

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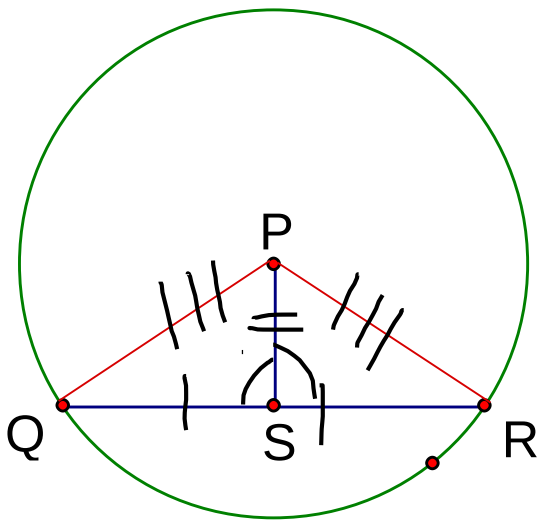


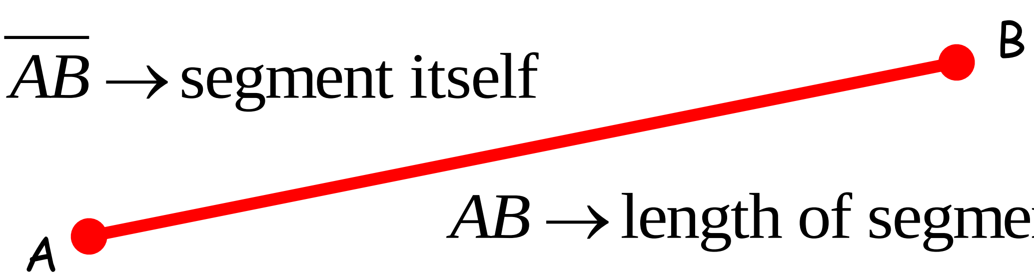
Statements	Reasons
1. Circle P	1. Given
2. S is the midpoint of $\overline{QR}$	2. Given
3. $\overline{QS} \cong \overline{SR}$	3. Def of midpt
4. $\overline{PS} \cong \overline{PS}$	4. Reflexive
5. Draw $\overline{PQ}$ and $\overline{PR}$	5. Two points det a seg
6. $\overline{PQ} \cong \overline{PR}$	6. All radii of a circle are $\cong$
7. $\Delta PQS \cong \Delta PRS$	7. SSS
8. $\angle PSQ \cong \angle PSR$	8. CPCTC
9. $\angle PSQ$ and $\angle PSR$ are supp	9. Def of supp
10. $\angle PSQ$ and $\angle PSR$ are rt $\angle$'s	10. If two angles are supp and congruent, then they are right angles.
11. $\overline{PS} \perp \overline{QR}$	11. Def of perpendicular

INDEX CARD

A.) (def) Distance (between two objects) is the length of the shortest path joining them.

B.) (postulate) A line segment is the shortest path between two points.

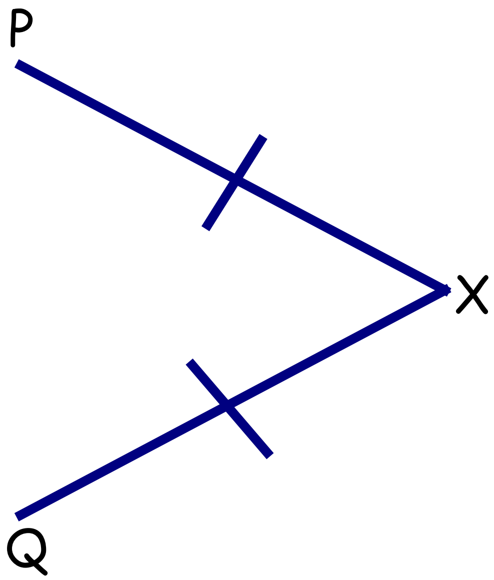
$\overline{AB} \rightarrow$ segment itself



$AB \rightarrow$ length of segment

Definition of Equidistance:

- If 2 points P and Q are the same distance from a third point X, then X is said to be EQUIDISTANT from P and Q.



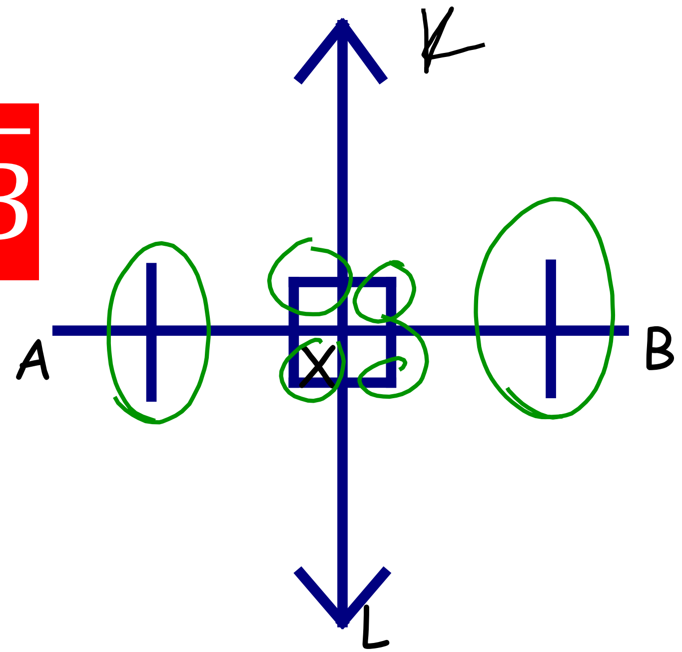
Another index card

Definition of a Perpendicular Bisector:

- A perpendicular bisector of a segment is the line that both **BISECTS** and is **PERPENDICULAR** to the segment.

L is $\perp$ bisector of $\overline{AB}$

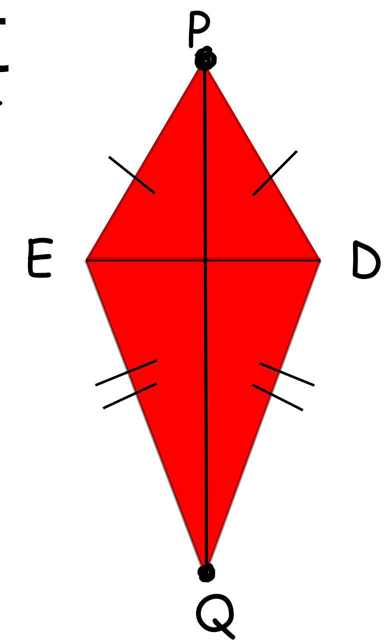
1) cuts in half
2) forms rt ∠'s



Index Card

Theorem: If 2 points are equidistant from the endpoints of a segment, then they determine the perpendicular bisector of that segment.

P and Q are two points that are *equidistant* from E and D (the endpoints of segment ED), so they determine the perpendicular bisector of the segment (ED).



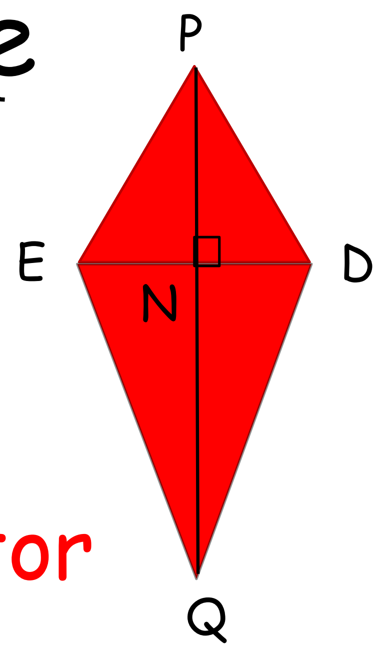
NEEDED: 2 points equidistant or

2 pairs congruent segments

Index Card

Theorem: If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

If N is on PQ, the perpendicular bisector of ED, then N is equidistant from E and D. (EN = ND)

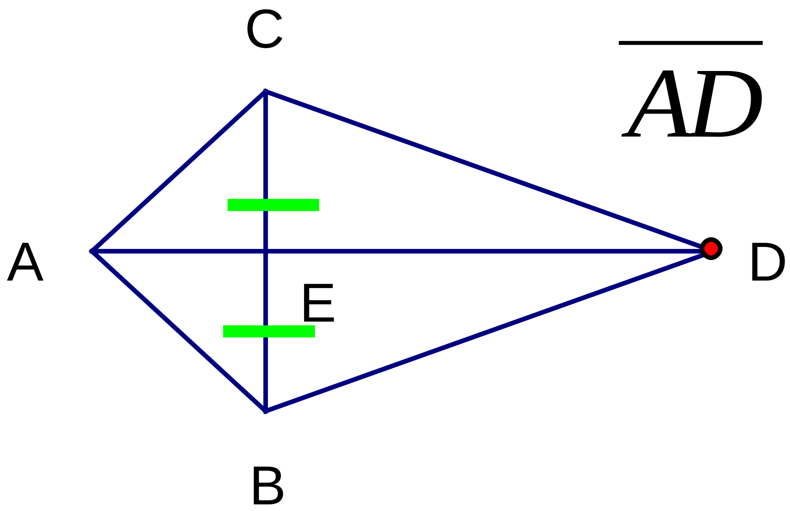


NEEDED: Perpendicular bisector of the segment

Index Card

TRUE/FALSE PRACTICE

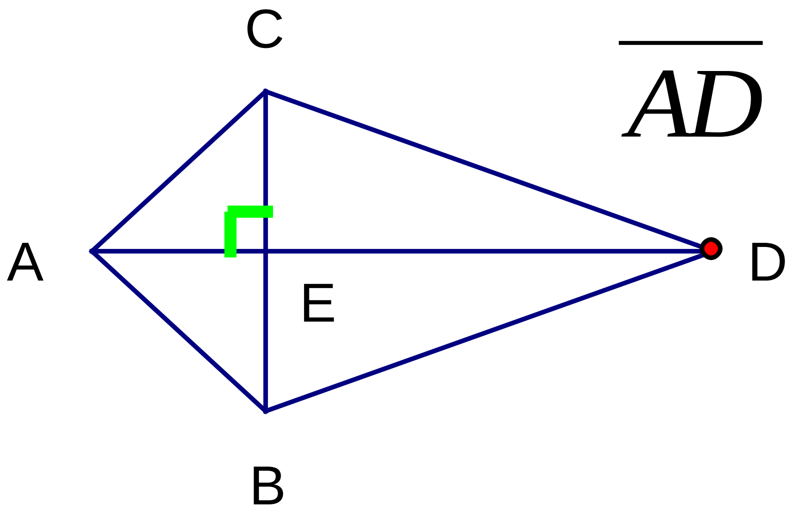
Ready??



$\overline{AD} \perp bisector\ of\ \overline{BC}$

E is the midpoint of BC.

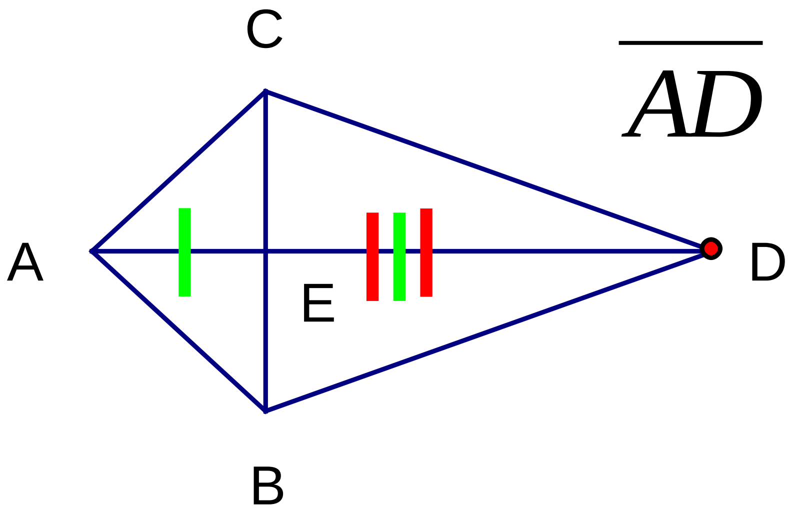
TRUE



$$\overline{AD} \perp bisector\ of\ \overline{BC}$$

<AEC is a right angle

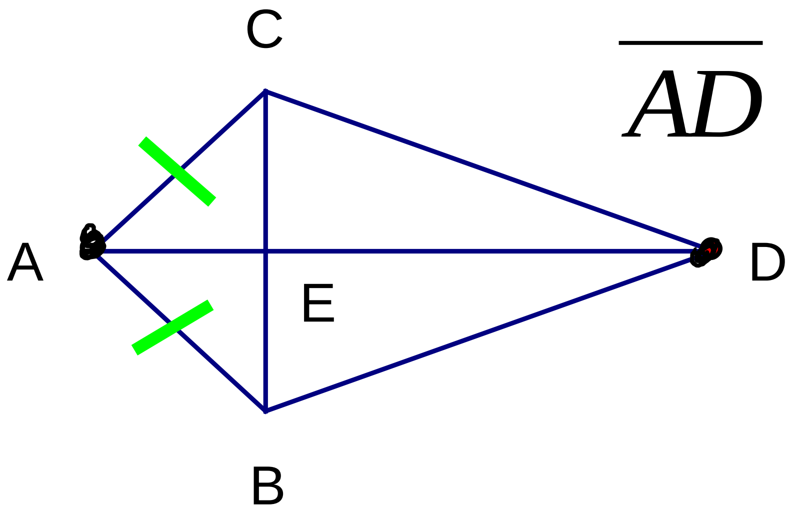
TRUE



$\overline{AD} \perp bisector\ of\ \overline{BC}$

E is the midpoint of AD

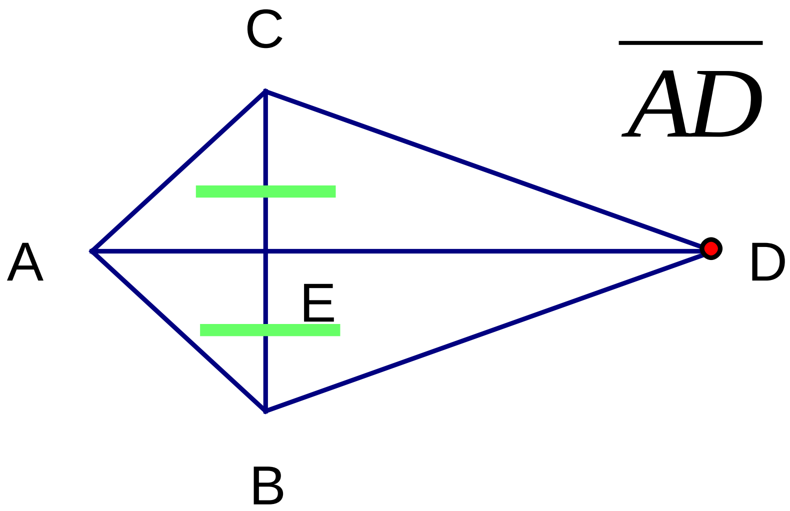
FALSE



$\overline{AD} \perp bisector\ of\ \overline{BC}$

$\overline{AC} \cong \overline{AB}$

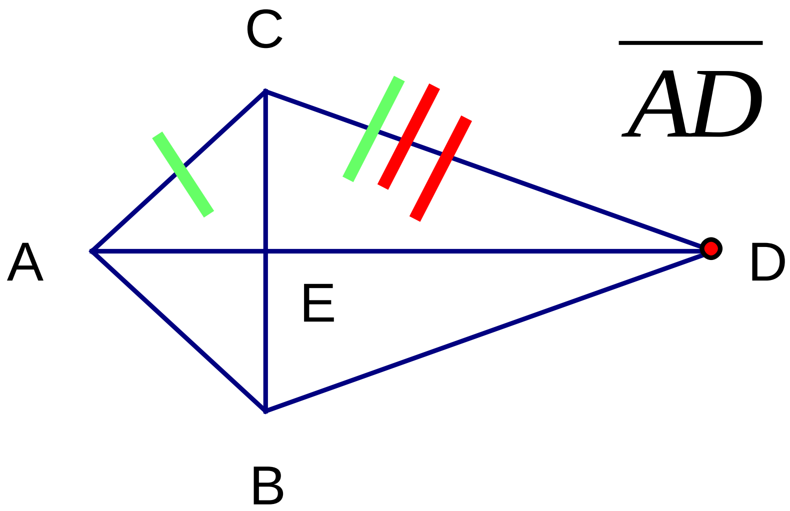
TRUE



$\overline{AD} \perp bisector\ of\ \overline{BC}$

$\overline{CE} \cong \overline{BE}$

TRUE

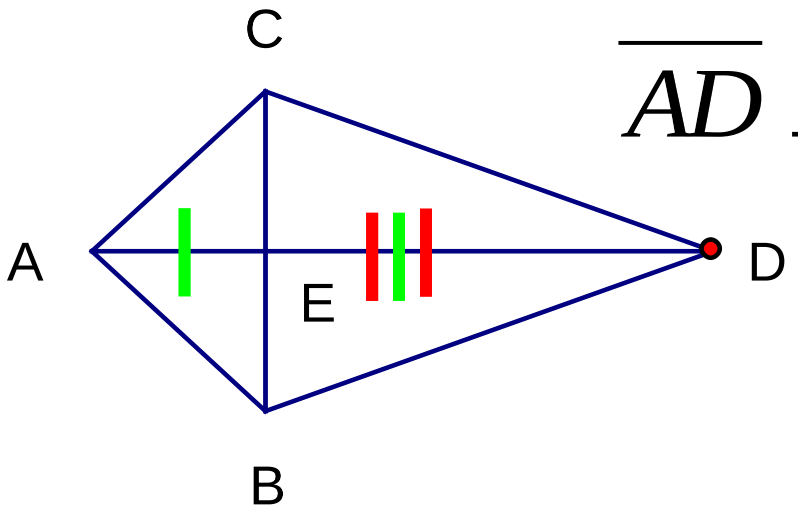


$\overline{AD} \perp bisector\ of\ \overline{BC}$

$\overline{CA} \cong \overline{CD}$

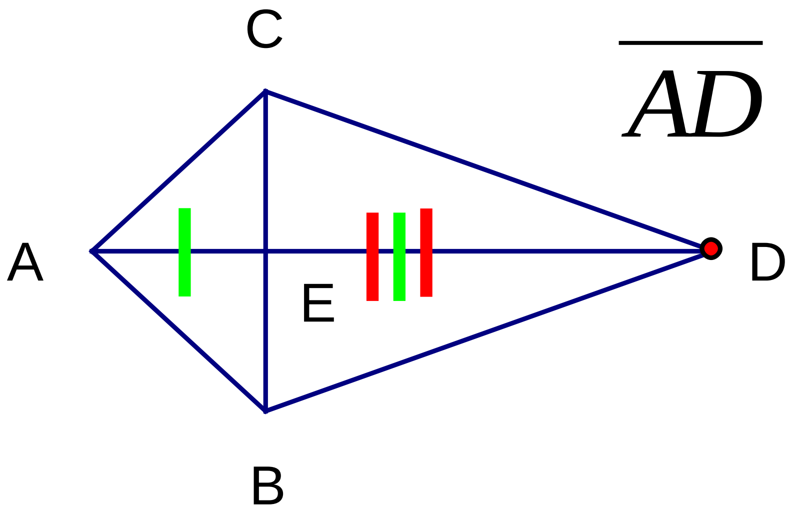
FALSE

$\overline{AD} \perp bisector\ of\ \overline{BC}$



$\overline{AE} \cong \overline{ED}$

FALSE

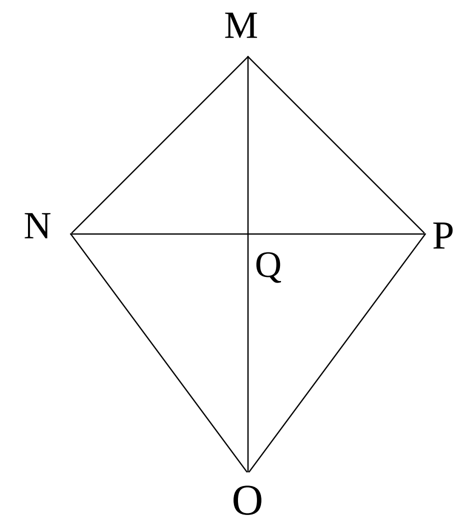


$\overline{AD} \perp bisector\ of\ \overline{BC}$

$\overline{CB}\ bisects\ \overline{AD}$

FALSE

Example #1



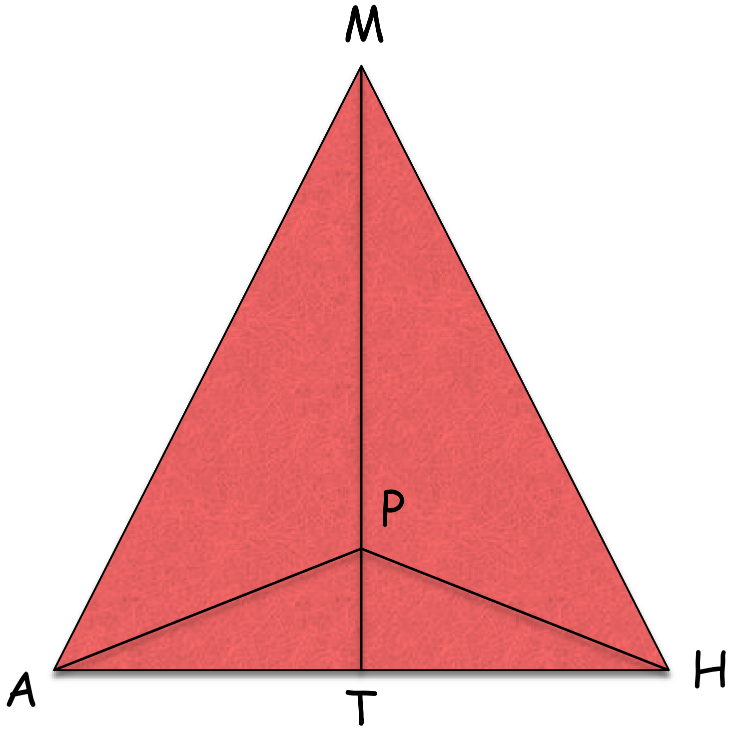
Given: MN ≅ MP

NQ ≅ PQ

Prove: NO ≅ PO

Statements	Reasons
1. MN ≅ MP	1. Given
2. NQ ≅ PQ	2. Given.
3. MQ is perpendicular to NP	3. If two points are equidistant from the endpoints of a segment, then they determine the perpendicular bisector of that segment.
4. NO = PO	4. If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of that segment.

Example Problem 2



Given: $\overline{AM} \cong \overline{MH}$

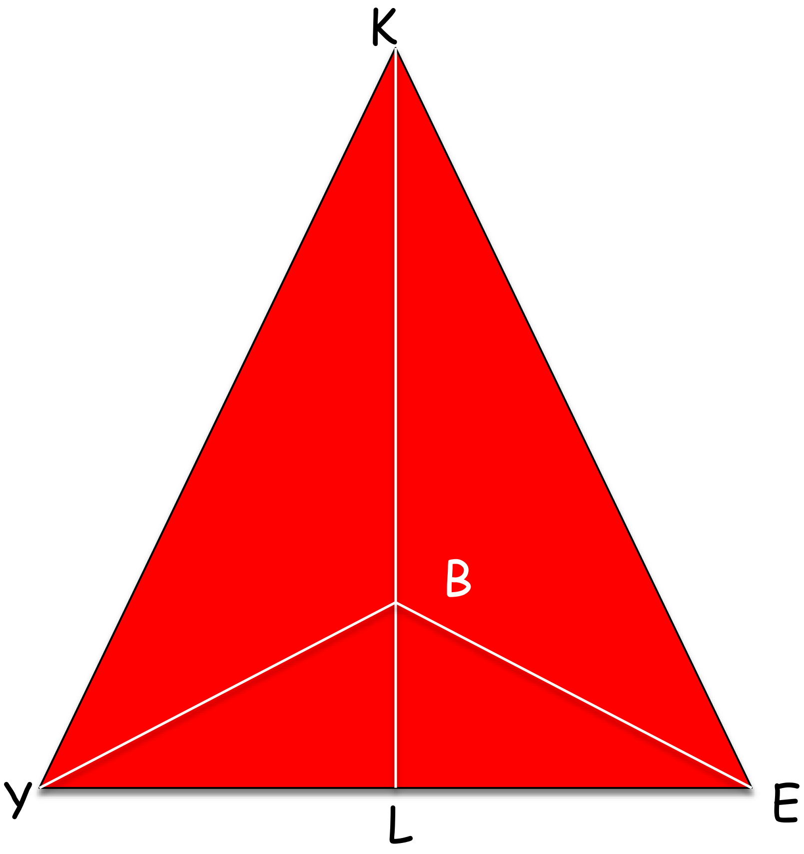
$\overline{AP} \cong \overline{PH}$

Prove: $\Delta APT \cong \Delta HPT$

EXAMPLE PROBLEM 2 SOLUTION

Statements	Reasons
1. $\overline{AM} \cong \overline{MH}$	1. Given
2. $\overline{AP} \cong \overline{PH}$	2. Given
3. $\overline{MT}$ is the perpendicular bisector of $\overline{AH}$	3. If two points are equidistant from the endpoints of a segment, then they determine the perpendicular bisector of that segment.
4. < PTH, <PTA are right angles	4. Perpendicular lines form right angles (Def of perpendicular)
5. ΔPTH, ΔPTA are right triangles	5. Def of right triangle
6. $\overline{PT} \cong \overline{PT}$	6. Reflexive
7. ΔAPT≅ΔHPT	7. HL (2,5,6)

Example Problem 3



Given: $\overline{KL}$ is the perpendicular bisector of $\overline{YE}$

Prove: $\Delta KBY \cong \Delta KBE$

EXAMPLE PROBLEM 3 SOLUTION

Statements	Reasons
1. $\overline{KL}$ is the perpendicular bisector of $\overline{YE}$	1. Given
2. $\overline{YB} \cong \overline{BE}$	2. If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of that segment
3. $\overline{KY} \cong \overline{KE}$	3. Same as 2
4. $\overline{KB} \cong \overline{KB}$	4. Reflexive
5. $\Delta KBY \cong \Delta KBE$	5. SSS (2,3,4)

Homework

- p. 182 #4 and 9, 11
- p. 187 #3 – 5, 7, 12, 14, 15