Theorems involving proportions in triangles

Section 8.4 and 8.5

Shadow Problem

- While waiting outside for a friend, Matthew noticed that the flagpole cast a 15-ft shadow and he himself cast a 4.5-ft shadow. If Matthew is 6 ft tall, how tall is the flagpole?

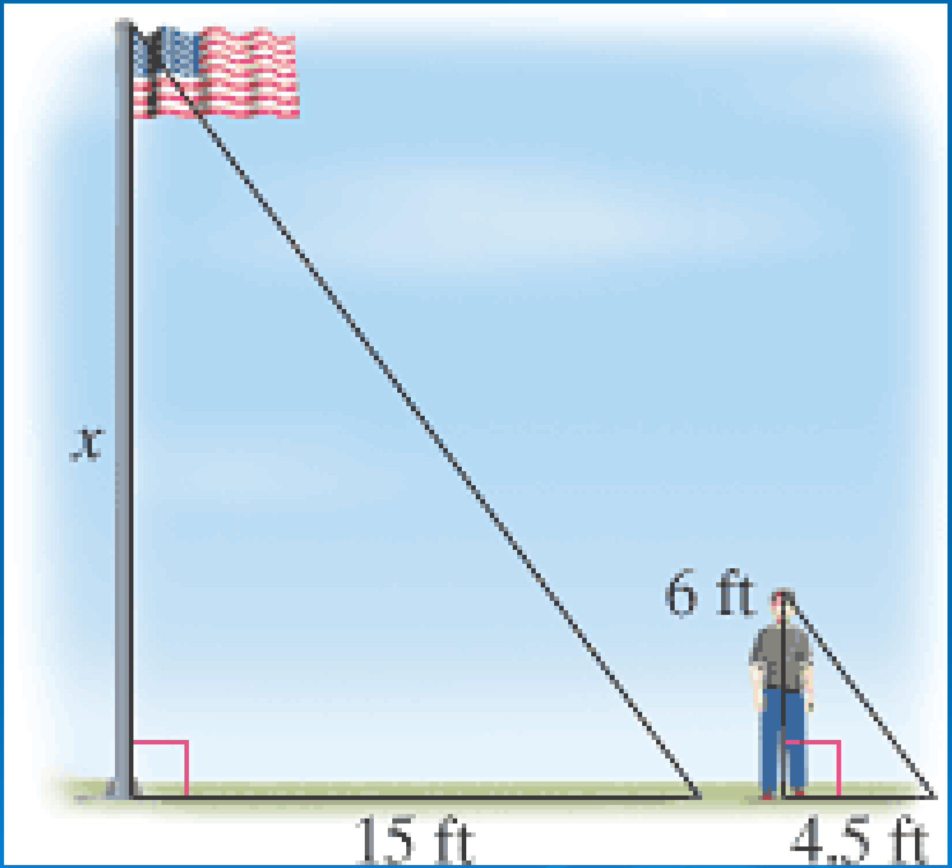
Not Drawn to scale

$$\frac{15}{4.5} = \frac{x}{6}$$

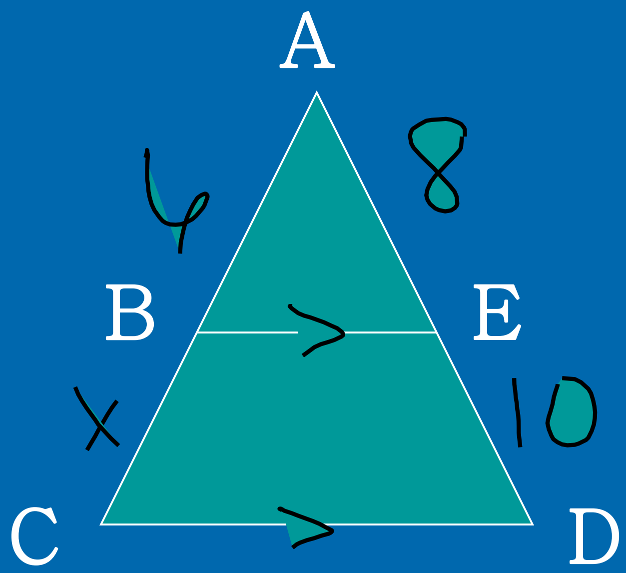
$$\frac{6}{4.5} = \frac{x}{15}$$

$$\cancel{\frac{x}{6} = \frac{4.5}{15}}$$

Solution



Index Card: Side-Splitter Theorem



If $\overleftrightarrow{BE} \parallel \overleftrightarrow{CD}$

Then $\dfrac{AB}{BC} = \dfrac{AE}{ED}$

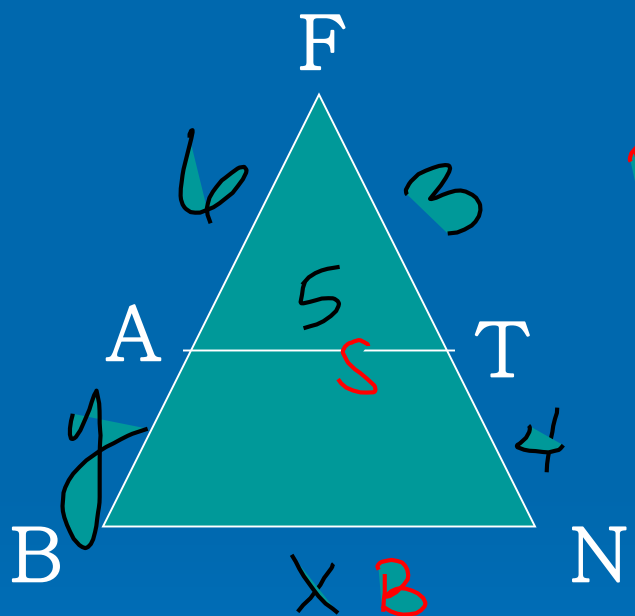
Theorem: If a line is || to one side of a Δ and intersects the other 2 sides, it divides those 2 sides proportionally. (Side-Splitter Theorem)

Given: $\overline{AT} \parallel \overline{BN}$

$FA = 6,\ FT = 3,\ AT = 5,\ TN = 4$

Find: length BN and AB

Example 1



You try this one!

Remember to label the shape and set up proportions using corresponding sides or side-splitter theorem.

$3x = 35$

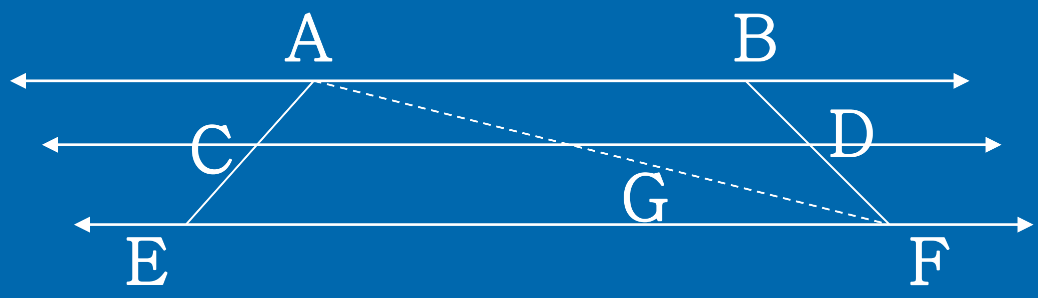
$x = 11\frac{2}{3}$

Answer:

- BN = 35/3 or 11 2/3
- AB = 8

Did you get it?

Theorem: If three or more parallel lines are intersected by two transversals, the parallel lines divide the transversals proportionally.



Index Card

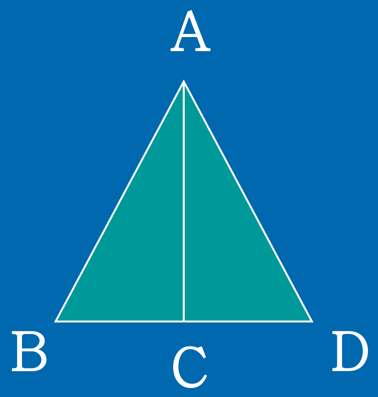
Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

Conclusion: $\dfrac{AC}{CE} = \dfrac{BD}{DF}$

$\dfrac{AE}{CE} = \dfrac{BF}{DF}$

Index Card

Angle Bisector Theorem: If a ray bisects an angle of a triangle, it divides the opposite side into segments that are proportional to the adjacent sides.

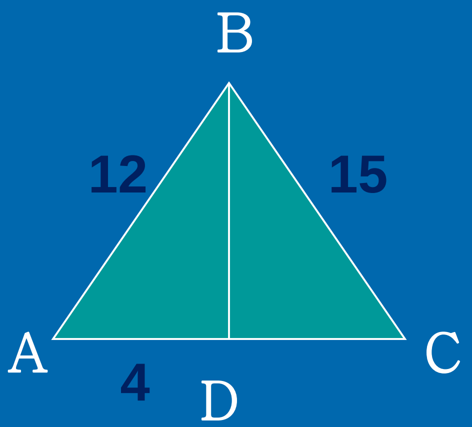


If ray AC bisects angle BAD, then $\frac{BC}{CD} = \frac{AB}{AD}$

L L

R R

$$\frac{BC}{AB} = \frac{CD}{AD}$$



Given: $\angle ABD \cong \angle DBC$

Lengths as shown

Find: DC

Example 2

$$\frac{4}{x} = \frac{12}{15}$$

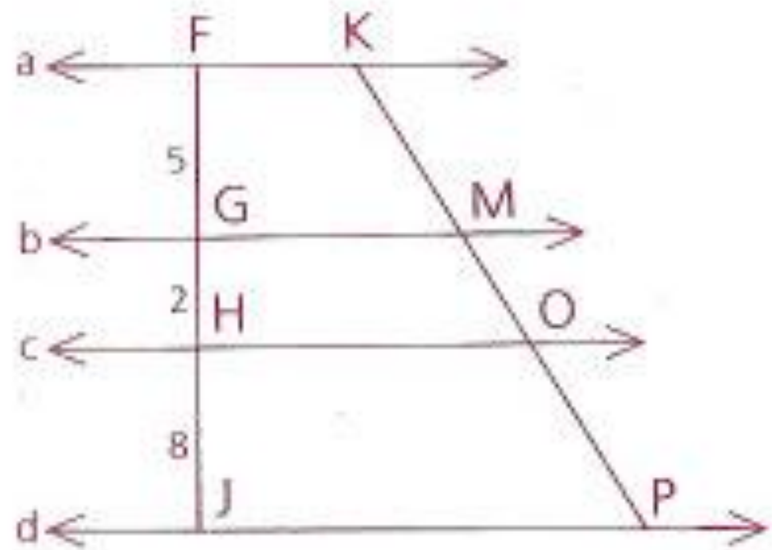
$x = 5$

$$\frac{12}{4} \qquad \frac{15}{x} \qquad \Bigg| \qquad \frac{12}{15} = \frac{4}{x}$$

Example 3

Given: a ∥ b ∥ c ∥ d,
lengths as shown,
KP = 24

Find: KM



According to Theorem 66, the ratio KM:MO:OP is equal to 5:2:8. Therefore, we let KM = 5x, MO = 2x, and OP = 8x. Since KP = 24,

$$5x + 2x + 8x = 24$$
$$15x = 24$$
$$x = \frac{24}{15} = \frac{8}{5}$$

Thus, KM = $5\left(\frac{8}{5}\right) = 8$

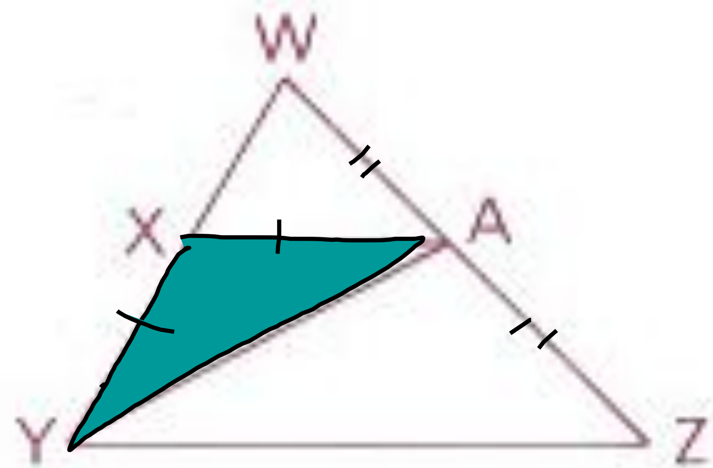
$\frac{15}{5} = \frac{24}{x}$ $\frac{15}{24} = \frac{5}{x}$

Example 4

Given: $\overleftrightarrow{XA} \parallel \overleftrightarrow{YZ}$,
$\angle XAY \cong \angle XYA$

Conclusion: $\dfrac{WX}{XA} = \dfrac{WA}{AZ}$

$\dfrac{WA}{AZ} = \dfrac{WX}{XY}$

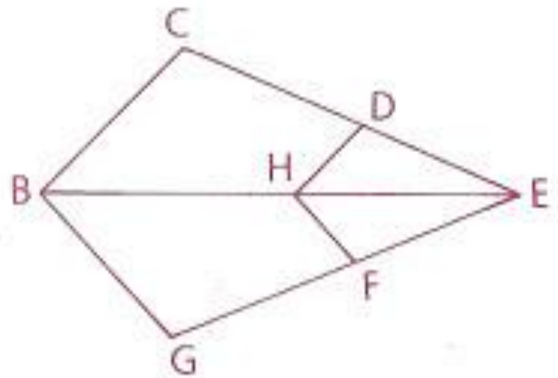


1 $\overleftrightarrow{XA} \parallel \overleftrightarrow{YZ}$	1 Given
2 $\dfrac{WX}{XY} = \dfrac{WA}{AZ}$	2 Side-Splitter Theorem
3 $\angle XAY \cong \angle XYA$	3 Given
4 $\overline{XA} \cong \overline{XY}$	4 If $\triangle$, then $\triangle$.
5 $\dfrac{WX}{XA} = \dfrac{WA}{AZ}$	5 Substitution (4 in 2)

Example 5

Given: $\overleftrightarrow{DH} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{HF} \parallel \overleftrightarrow{BG}$

Prove: $\frac{CD}{DE} = \frac{GF}{FE}$



1 $\overleftrightarrow{DH} \parallel \overleftrightarrow{BC}$	1 Given
2 $\frac{CD}{DE} = \frac{BH}{HE}$	2 Side-Splitter Theorem
3 $\overleftrightarrow{HF} \parallel \overleftrightarrow{BG}$	3 Given
4 $\frac{BH}{HE} = \frac{GF}{FE}$	4 Same as 2
5 $\frac{CD}{DE} = \frac{GF}{FE}$	5 Transitive Property (2, 4)

Partner Problems

- Work with your partner to complete
  - p. 348 #9, 11, 17 – 21
  - p. 355 #2, 5 – 7, 11, 13, 14, 20
- Refer to your index cards if needed!!!

Homework

- Watch these videos:
  - Similarity Example Where Same Side Plays Different Roles
  - Similar Right Triangle Examples
  - Review of Radicals