9.3
Altitude-On-Hypotenuse Theorems

(a.k.a Geometry Mean)

Warm Up

1, 4, 9, 16, 25, 36, 49, . . .

Simplify.

1. $\sqrt{150}$

$5\sqrt{3 \cdot 2}$

$5\sqrt{6}$

150 → 50 · 3

50 → 25 · 2

25 → 5 · 5

2. $5\sqrt{32}$

$5\sqrt{16 \cdot 2}$

$5 \cdot 4\sqrt{2}$

$20\sqrt{2}$

3. $\left(2\sqrt{3}\right)^2$

$4 \cdot \sqrt{3}^2$

$4 \cdot 3$

12

Given two numbers a and b, x is the geometric mean if

EXTREMES

$$\frac{a}{x} = \frac{x}{b}$$

MEANS

Index Card

Find the geometric mean of 9 and 16

$$\frac{a}{x} = \frac{x}{b}$$

$$\frac{9}{x} = \frac{x}{16}$$

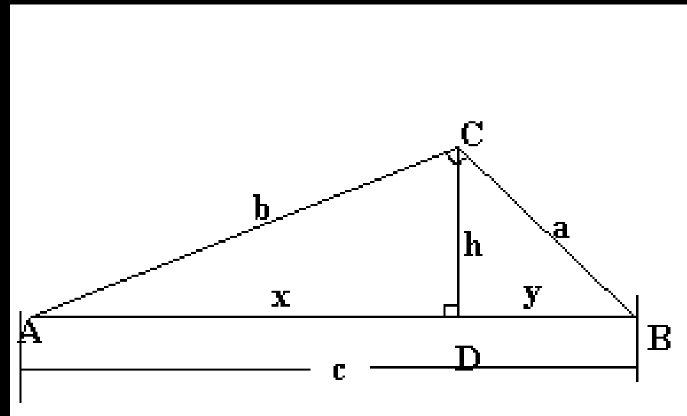
$$x^2 = 144$$

$$x = \pm 12$$

$\frac{9}{x} = \frac{x}{16}$

$\sqrt{x^2} = \sqrt{144}$

$x = 12$

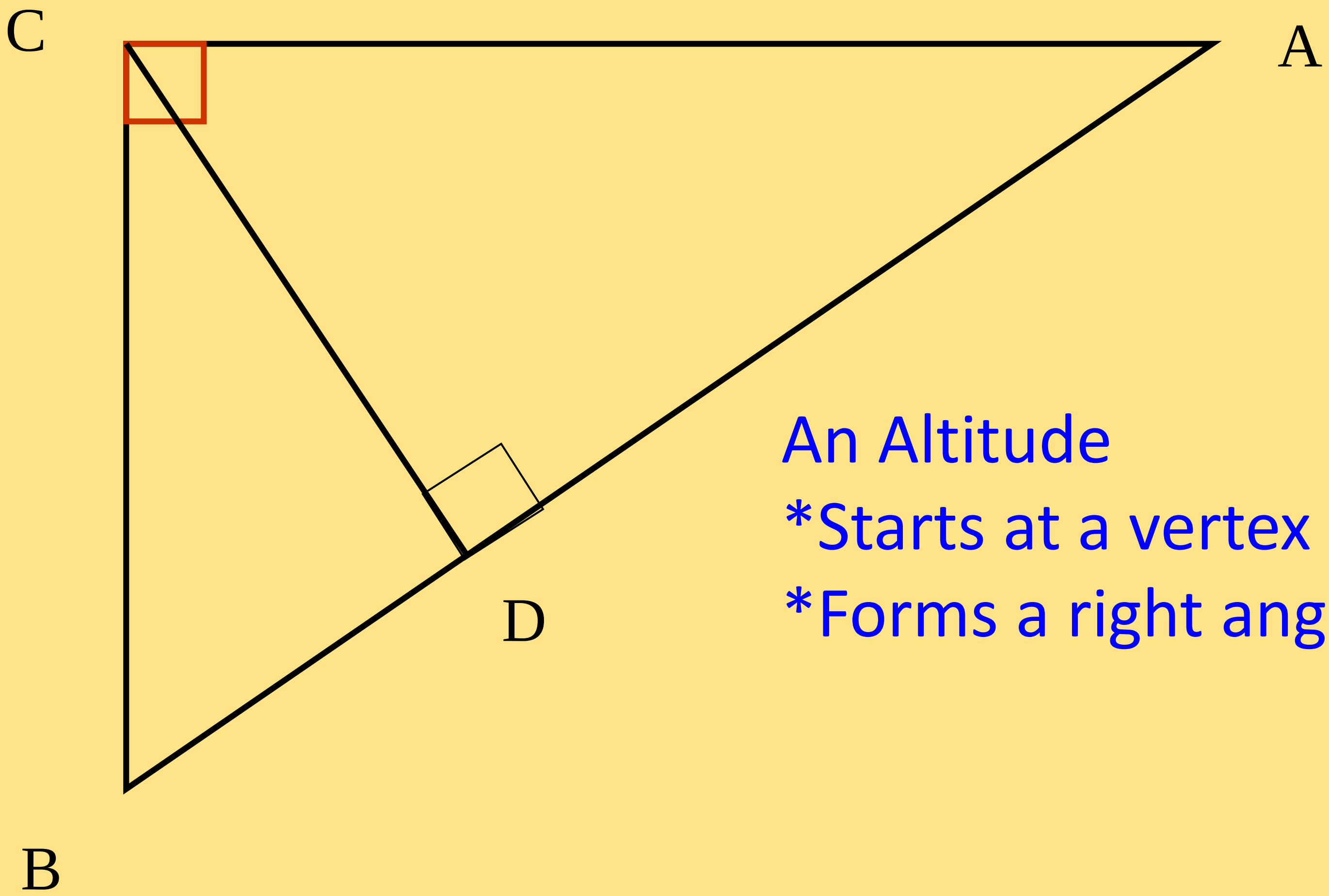


Index Card: When an altitude is drawn from a vertex to a hypotenuse, then three similar triangles are formed.

**Identify the three similar triangles! Which theorem can they be proven similar?

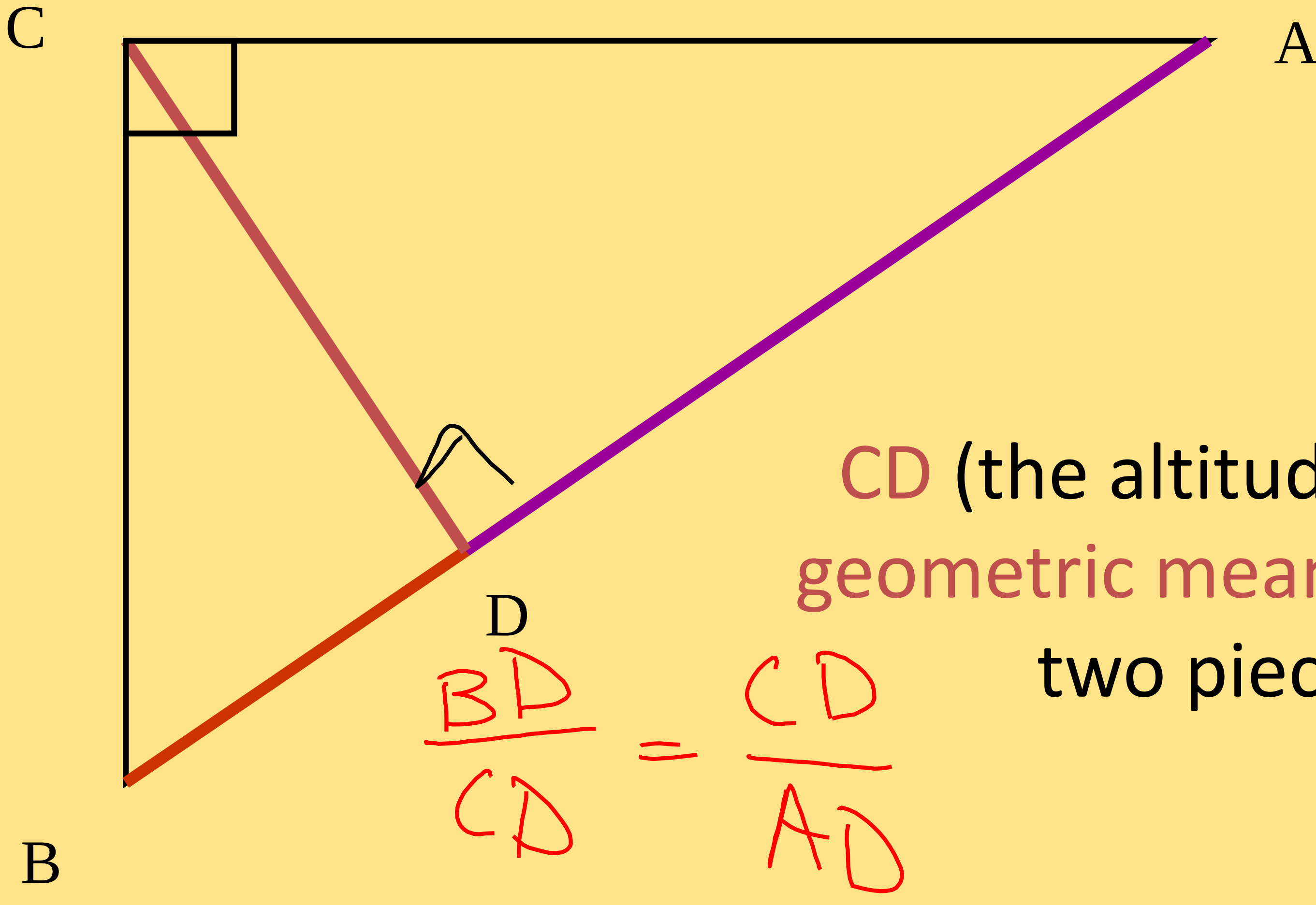
Right $\triangle$s

Altitudes!!!!
What do you remember?



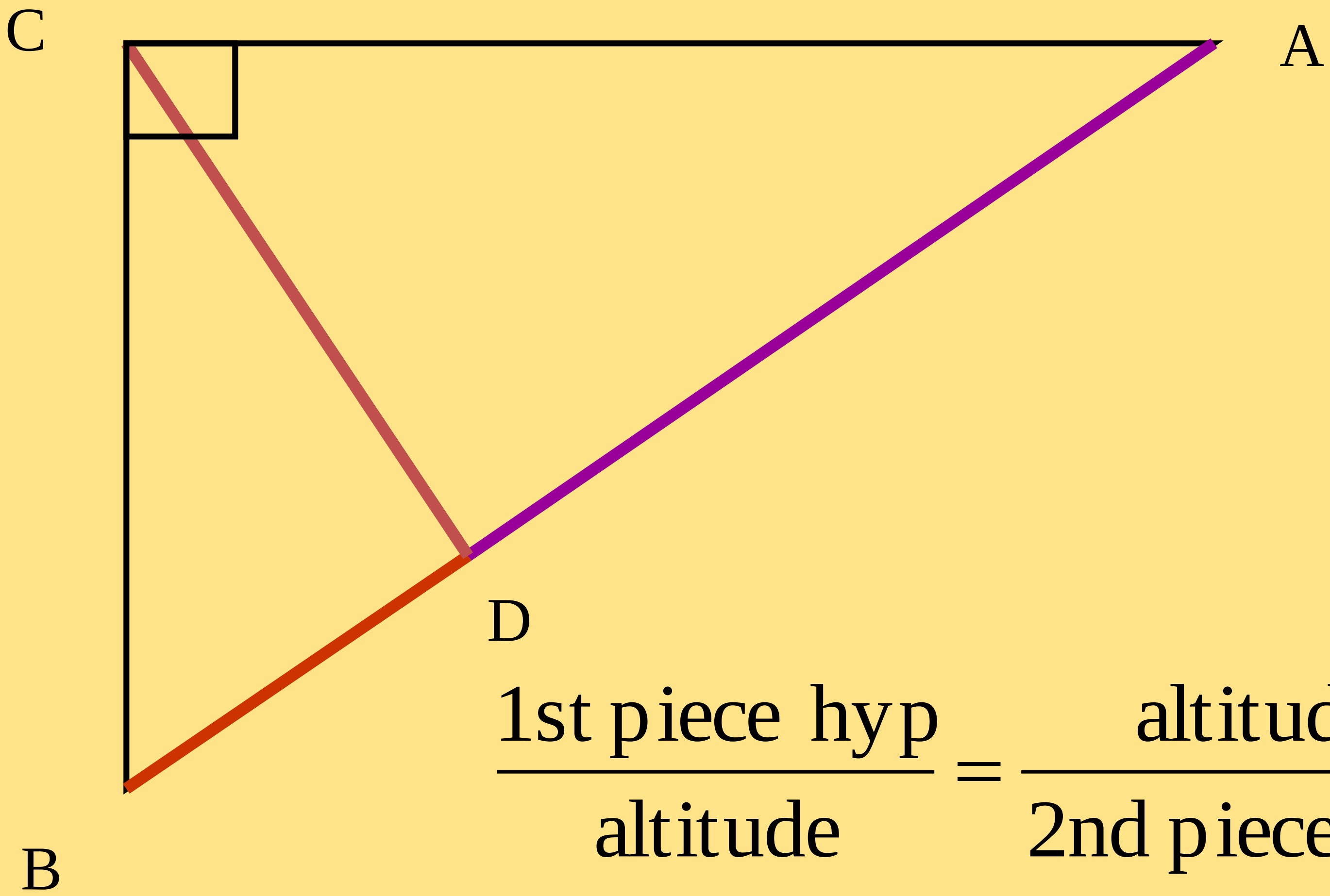
An Altitude
*Starts at a vertex
*Forms a right angle

The hypotenuse is split into two pieces BD and DA



CD (the altitude) is the geometric mean of those two pieces

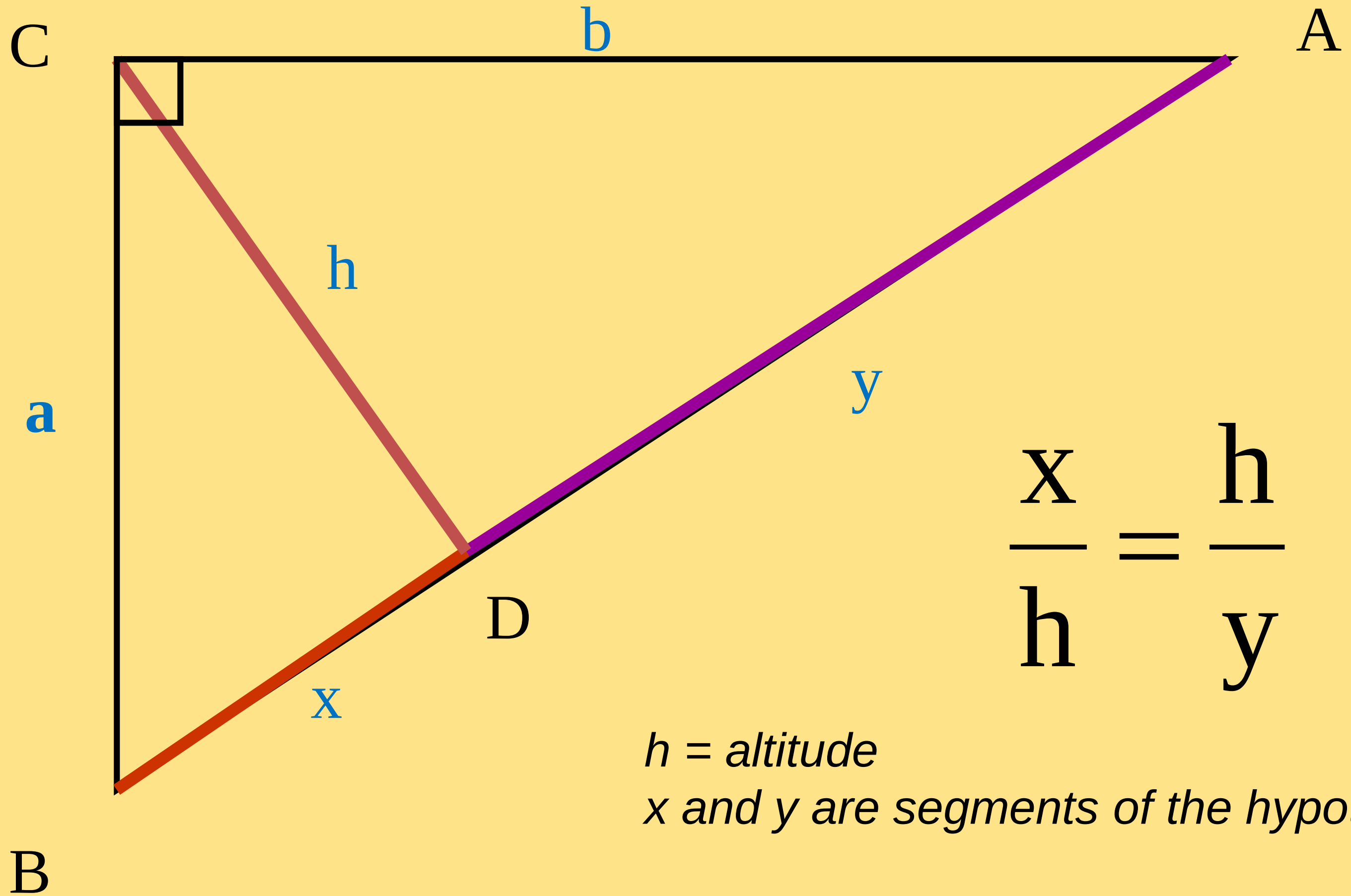
If the altitude is in the means place, put the two segments of the hypotenuse into the extremes.



$$\frac{\text{1st piece hyp}}{\text{altitude}} = \frac{\text{altitude}}{\text{2nd piece hyp}}$$

Index Card

Theorem: The altitude to the hypotenuse is the mean proportional (or geometric mean) between the segments of the hypotenuse.

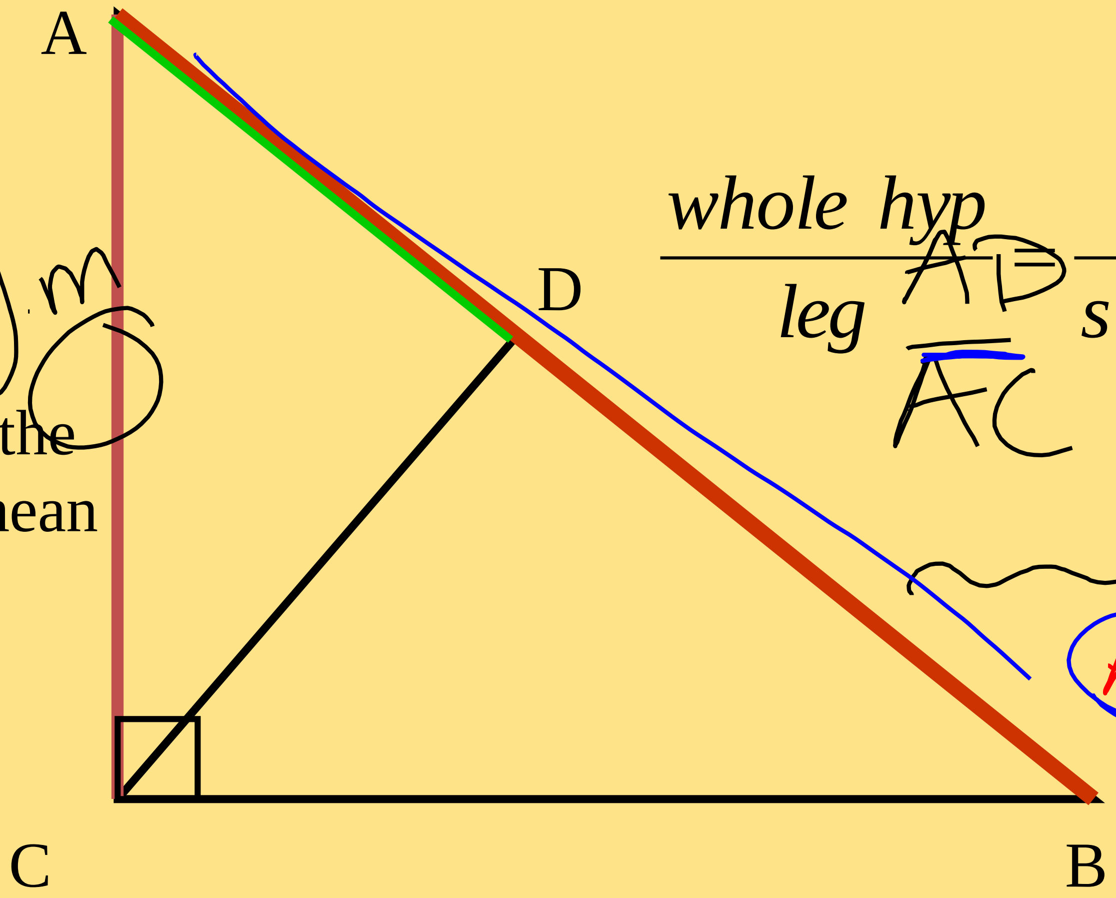


$$\frac{x}{h} = \frac{h}{y}$$

h = altitude

x and y are segments of the hypotenuse

If the leg is in the means place, put the whole hyp. and segment adjacent to that leg into the extremes.



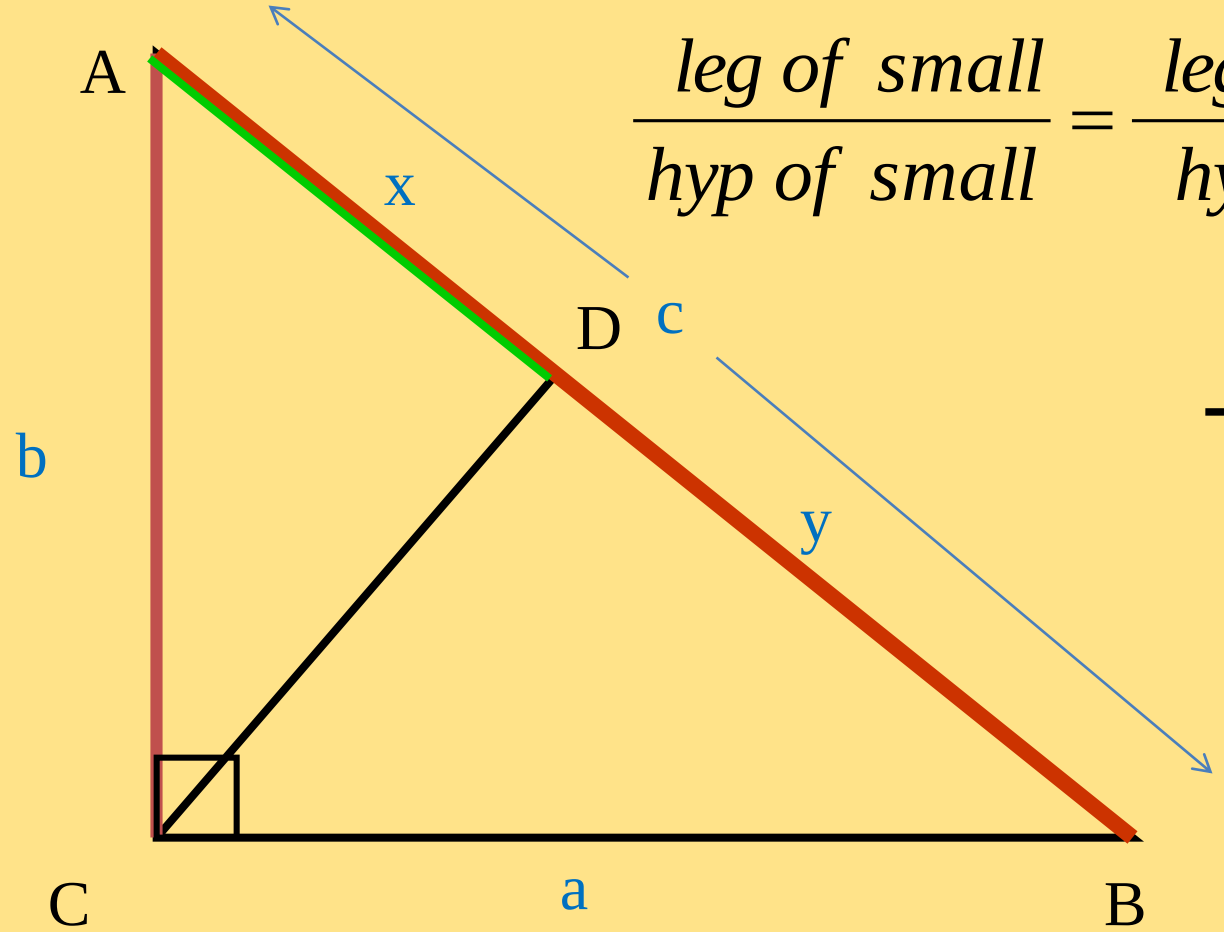
the LEG is the geometric mean

$$\frac{whole\ hyp}{leg} \qquad \frac{leg}{side\ adj\ to\ leg}$$

$$\frac{AB}{AC} = \frac{AC}{AD}$$

Index Card

Or use similar triangles…using the large triangle and the small triangle, set up a proportion using the hypotenuse and the short leg.



$$\frac{leg\ of\ small}{hyp\ of\ small} = \frac{leg\ of\ large}{hyp\ of\ large}$$

$$\frac{x}{b} = \frac{b}{c}$$

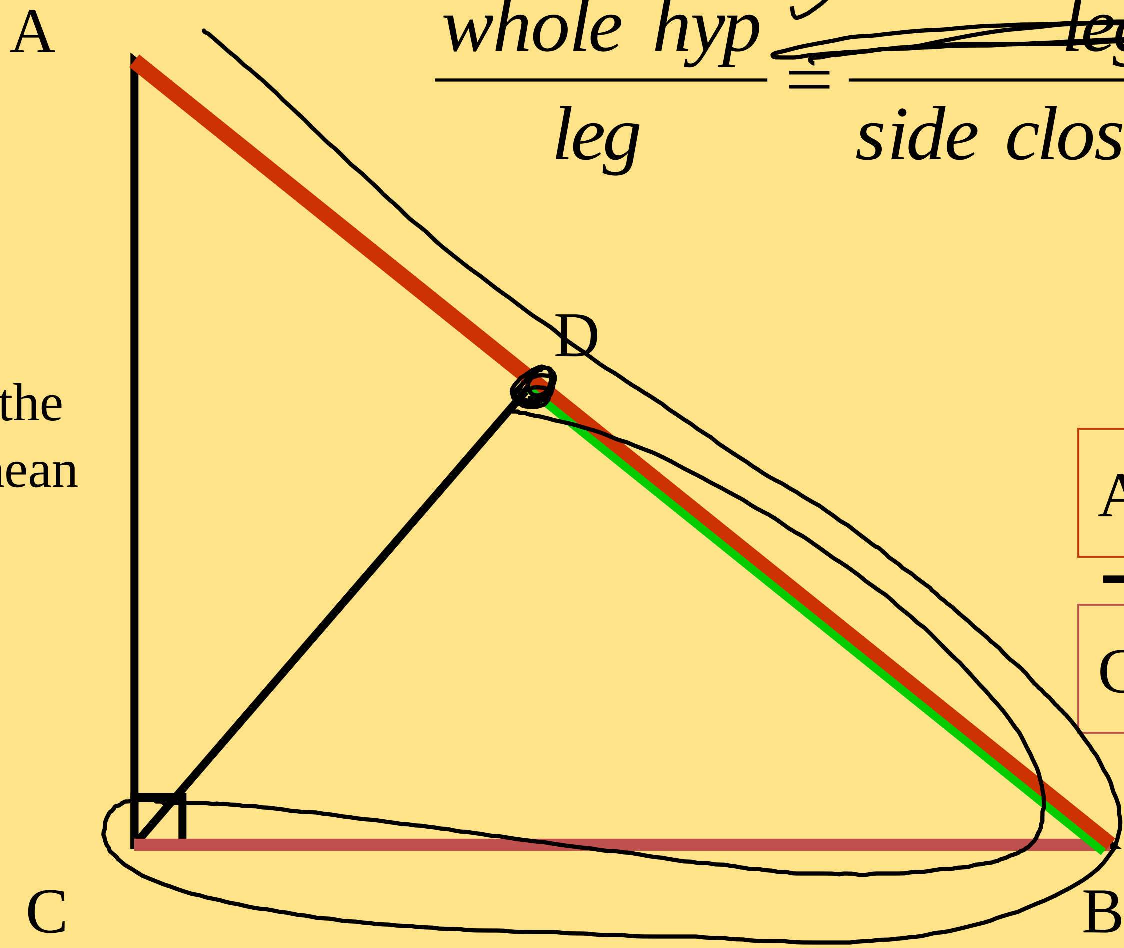
Index Card

Index Card

Using the other part of the hypotenuse:

Boomerang

$$\frac{\text{whole hyp}}{\text{leg}} = \frac{\text{leg}}{\text{side close to leg}}$$



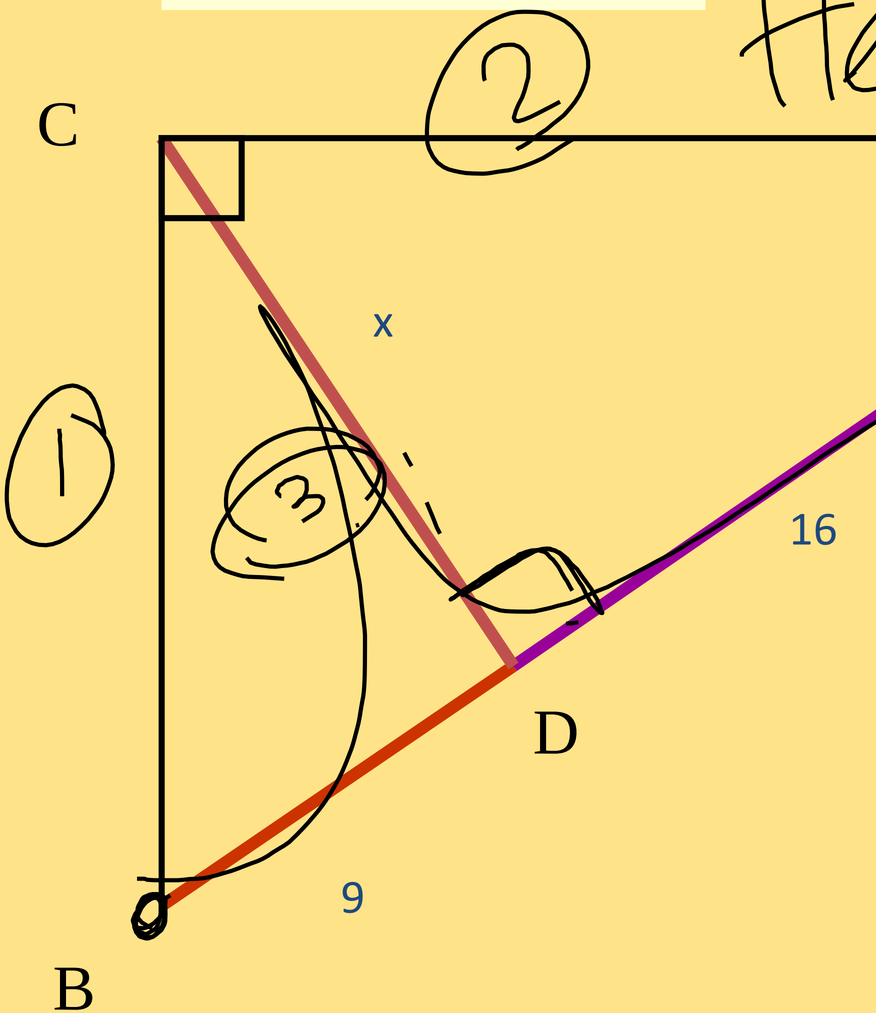
the LEG is the geometric mean

$$\frac{AB}{CB} = \frac{CB}{BD}$$

Remember: ***You can use similar triangles…compare the hypotenuse and the long leg using the large triangle and the medium-sized triangle.***

EX: 1 Find x

Heartbeat



$$\frac{9}{x} = \frac{x}{16}$$

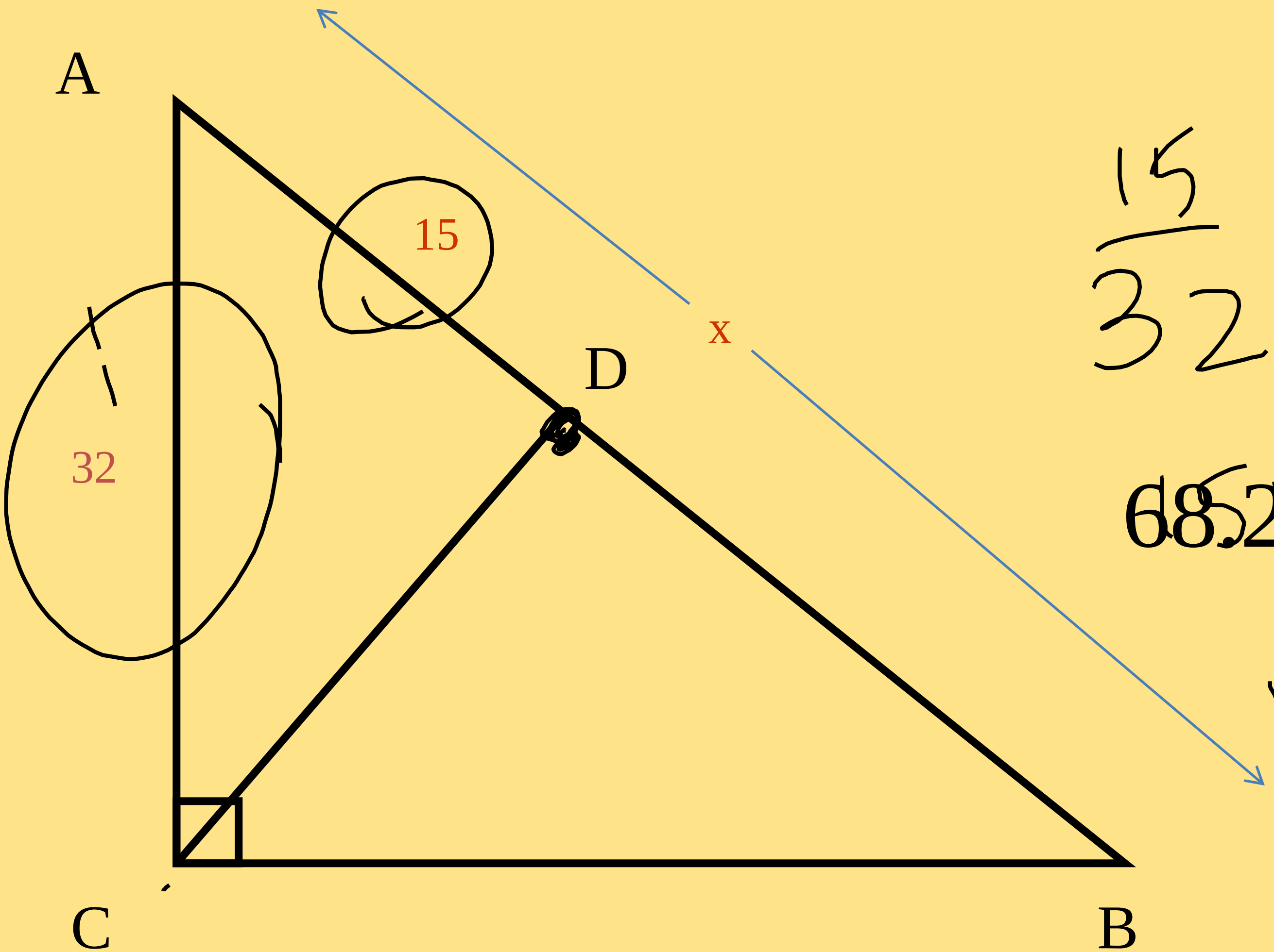
$$x^2 = 144$$

$$\sqrt{x^2} = \sqrt{144}$$

$$x = 12$$

$x = 12$

EX: 2 Find the length of AB.

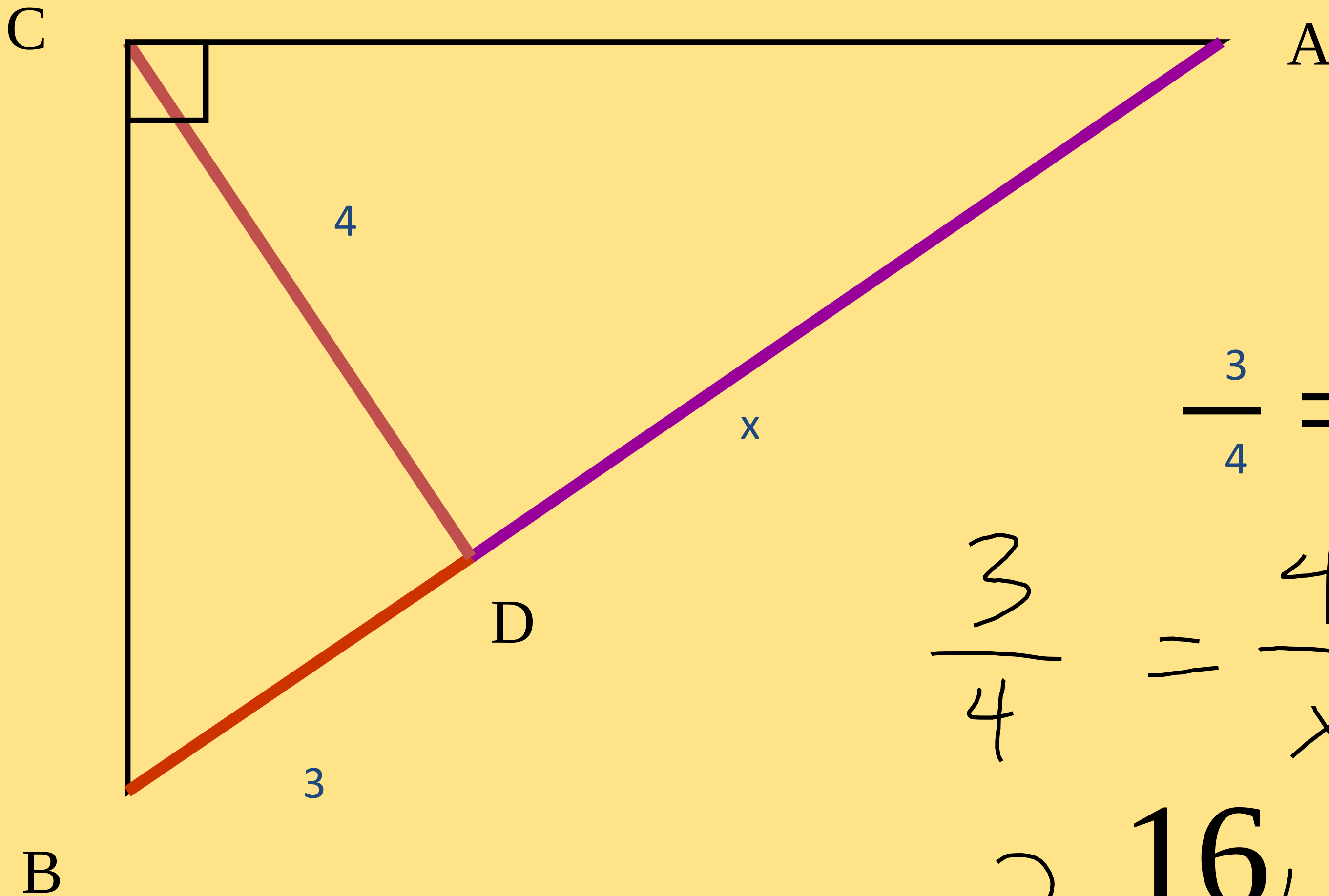


$$\frac{15}{32} = \frac{32}{x}$$

$$68.26 \approx \frac{1024}{15} = x$$

$x \approx 68.3$

EX 3: Solve for x



$$\frac{3}{4} = \frac{4}{x}$$

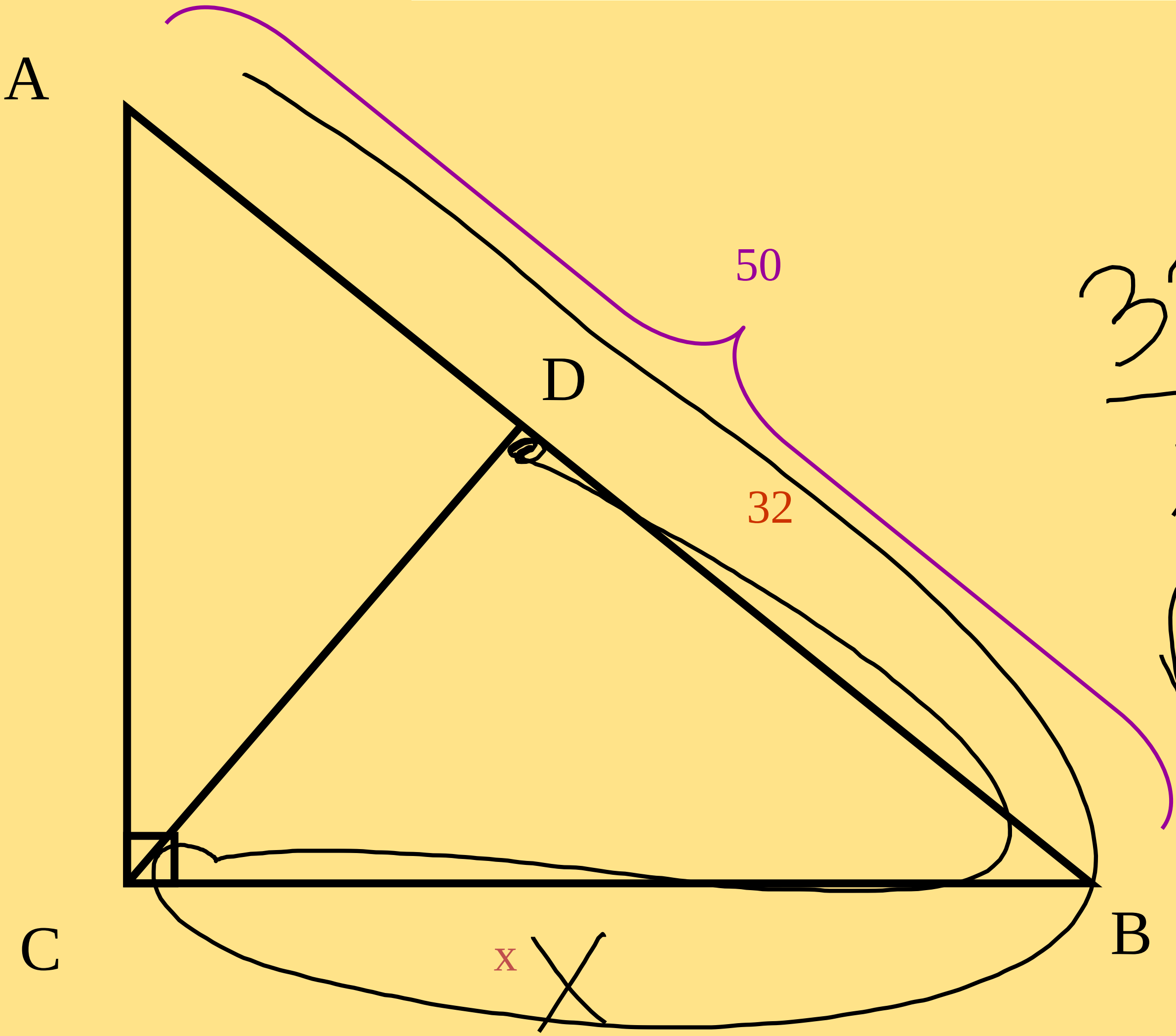
$$\frac{3}{4} = \frac{4}{x}$$

$$x = \frac{16}{3} \approx 5.33$$

$$3x = 16$$

$$x = 5.\overline{3}$$

EX: 4 Find the length of CB.

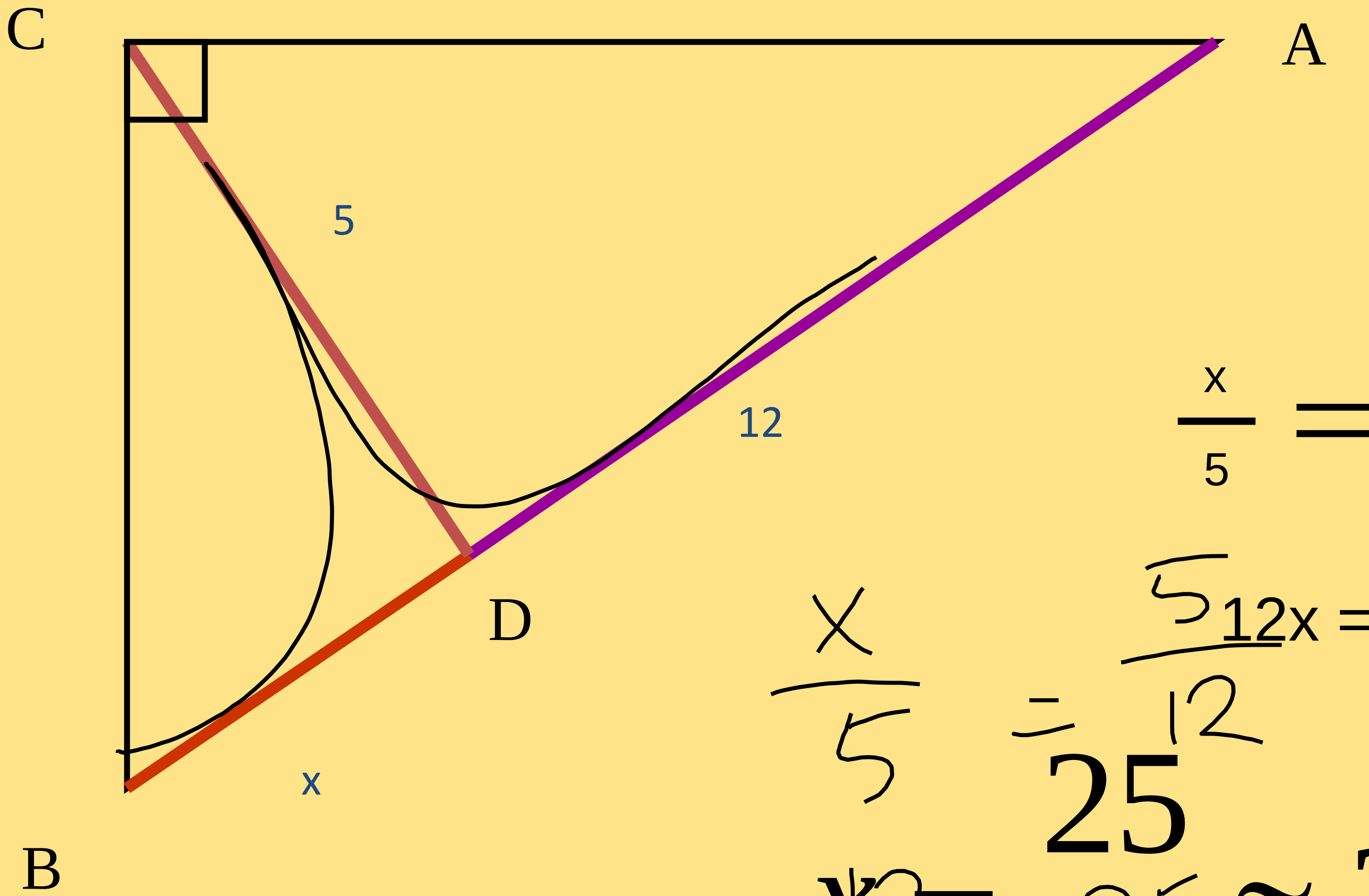


$$\frac{32}{x} = \frac{x}{50}$$

$$x^2 = 1600$$

$$x = 40$$

EX: 5 Solve for x



$$\frac{x}{5} = \frac{5}{12}$$

$$12x = 25$$

$$x = \frac{25}{12} \approx 2.08$$

Homework

- p. 379 #1-5, 16, 17